

# ON CONVOLUTION FOR GENERAL NOVEL FRACTIONAL WAVELET TRANSFORM

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ABSTRACT. Using Pathak and Pathak techniques, the basic function  $D^\alpha(u, v, w)$  associated with general novel fractional wavelet transform (GNFrWT) is defined and its properties are investigated. By using basic function  $D^\alpha(u, v, w)$  translation and convolution associated with GNFrWT are defined and certain existence theorems are proved for basic function and associated convolution.

## 1. INTRODUCTION

The general novel fractional wavelet transform (GNFrWT) of a function  $h(t)$  with respect to wavelet  $\phi$  can be defined as [1],

$$(W_\phi^\alpha h)(a, b) = \int_{\mathbb{R}} h(t) \overline{\phi_{a,b}^\alpha(t)} dt, \quad (1.1)$$

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2000 *Mathematics Subject Classification.* 22E46, 53C35, 57S20.

*Key words and phrases.* Fractional fourier transform; novel fractional wavelet transform.

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where

$$\phi_{a,b}^\alpha(t) = e^{(-i/2)(t^2-b^2)\cot\theta} |a|^{-\rho} \phi\left(\frac{t-b}{a}\right); \quad a \in \mathbb{R}_+, \quad b \in \mathbb{R} \text{ and } \rho \geq 0. \quad (1.2)$$

The inversion formula for (1.1) with respect to the  $e^{(-i/2)(t^2-b^2)\cot\theta} \phi_{a,b}(t)$  can be given as

$$h(t) = \frac{1}{c_\phi} \int_{\mathbb{R}} \int_{\mathbb{R}} (W_\phi^\alpha h)(a, b) \phi_{a,b}^\alpha(t) |a|^{2\rho-3} db da, \quad (1.3)$$

where

$$O < C_\phi = \int_{\mathbb{R}} \frac{|\hat{\phi}(\omega)|^2}{|\omega|} d\omega < \infty. \quad (1.4)$$

If  $\alpha = 1$ , the GNFrWT, correlate with the wavelet transform (WT). As [1], the GNFrWT can be expressed as in terms of the fractional fourier transform  $H^\alpha(v)$  of the signal  $h(t)$ .

$$(W_h^\alpha)(a, b) = \int_{\mathbb{R}} \sqrt{2\pi} a^{-\rho+1} H^\alpha(v) \overline{\hat{\phi}(av \csc \theta)} \overline{K_{-\alpha}(v, b)} dv \quad (1.5)$$

where  $\hat{\phi}(av \csc \theta)$  indicates the fourier transform of  $\phi(t)$ . Also, the GNFrWT can be rewritten as [1]

$$(W_\phi^\alpha h)(a, b) = e^{(-i/2)b^2 \cot \theta} \int_{\mathbb{R}} h(t) e^{(i/2)t^2 \cot \theta} \overline{\phi_{a,b}(t)} dt. \quad (1.6)$$

**Theorem 1.1.** (*Parseval formula*). If wavelet  $\phi \in L^2(\mathbb{R})$  and  $(W_\phi^\alpha h)(a, b)$  is the GNFrWT of  $h \in L^2(\mathbb{R})$ , then for any function  $h, g \in L^2(\mathbb{R})$ ,

$$\int_{\mathbb{R}} \int_{\mathbb{R}} (W_\phi^\alpha h)(a, b) \overline{(W_\phi^\alpha g)(a, b)} |a|^{2\rho-3} db da = C_\phi(h, g). \quad (1.7)$$

where

$$O < C_\phi = \int_{\mathbb{R}} \frac{|\hat{\phi}(\omega)|^2}{|\omega|} d\omega < \infty.$$

**Proof 1.** By using (1.5), we have

$$W_\phi^\alpha(h)(a, b) = \int_{\mathbb{R}} \sqrt{2\pi} a^{-\rho+1} H^\alpha(h)(u) \overline{\hat{\phi}(au \operatorname{cosec} \theta)} \overline{K_{-\alpha}(u, b)} du \quad (1.8)$$

$$\overline{W_\phi^\alpha(g)(a, b)} = \int_{\mathbb{R}} \sqrt{2\pi} a^{-\rho+1} \overline{H^\alpha(g)(v)} \hat{\phi}(av \operatorname{cosec} \theta) K_{-\alpha}(v, b) dv \quad (1.9)$$

from (1.8) and (1.9), we get

$$\int_{\mathbb{R}} \int_{\mathbb{R}} W_\phi^\alpha(h)(a, b) \overline{W_\phi^\alpha(g)(a, b)} |a|^{2\rho-3} db da = \langle h, g \rangle C_\phi. \quad (1.10)$$

**Theorem 1.2.** (Inversion Formula) If wavelet  $\phi \in L^2(\mathbb{R})$  and  $(W_\phi^\alpha h)(a, b)$  is the GNFrWT of  $h \in L^2(\mathbb{R})$ , then the reconstruction of  $h$  is given by

$$h(u) = \frac{1}{C_\phi} \int_{\mathbb{R}} \int_{\mathbb{R}} (W_\phi^\alpha h)(a, b) \phi_{a,b}^\alpha(t) |a|^{2\rho-3} db da \quad (1.11)$$

**Proof 2.** By using (1.10) for  $h = g$ , then we can find (1.11).

This paper is arranged in the following manner :- In the next section, we define basic function  $D^\alpha(u, v, w)$ , translation and associated convolution for GNFrWT. In the last third section, we obtained and established the certain existence theorem and convolution theorem, by using Pathak and Pathak techniques [6]

## 2. BASIC FUNCTION, TRANSLATION AND ASSOCIATED CONVOLUTION FOR GNFrWT

Now, by using Pathak and Pathak techniques [6], we define the basic function  $D^\alpha(u, v, w)$ , translation  $\tau_u^\alpha$  and associated convolution  $\#^\alpha$  operators for GNFrWT.

The basic function  $D^\alpha(u, v, w)$  for (1.1) is define as

$$\begin{aligned} W_\phi^\alpha[D^\alpha(u, v, w)](a, b) &= \int_{\mathbb{R}} D^\alpha(u, v, w) \overline{\phi_{a,b}^\alpha(t)} dt \\ &= \overline{\psi_{a,b}^\alpha(w)} \overline{\chi_{a,b}^\alpha(v)}, \end{aligned} \quad (2.1)$$

where  $\psi^\alpha, \phi^\alpha$  and  $\chi^\alpha$  are three fractional wavelets satisfying certain conditions (1.2).

Now, by using (1.3) we get,

$$D^\alpha(u, v, w) = C_\phi^{-1} \int_{\mathbb{R}} \int_{\mathbb{R}} \overline{\psi_{a,b}^\alpha(w)} \overline{\chi_{a,b}^\alpha(v)} \phi_{a,b}^\alpha(u) |a|^{2\rho-3} da db. \quad (2.2)$$

The translation  $\tau_u^\alpha$  is defined as [6]

$$\begin{aligned} (\tau_u^\alpha h)(v) &= h^*(u, v) = \int_{\mathbb{R}} D^\alpha(u, v, w) h(w) dw \\ &= C_\phi^{-1} \int_{\mathbb{R}} \int_{\mathbb{R}} \int_{\mathbb{R}} \overline{\psi_{a,b}^\alpha(w)} \overline{\chi_{a,b}^\alpha(v)} \phi_{a,b}^\alpha(u) h(w) |a|^{2\rho-3} da db dw. \end{aligned}$$

The associated convolution is defined as

$$\begin{aligned} (h \#^\alpha g)(u) &= \int_{\mathbb{R}} h^*(u, v) g(v) dv \\ &= \int_{\mathbb{R}} \int_{\mathbb{R}} D^\alpha(u, v, w) h(w) g(v) dv dw \\ &= C_\phi^{-1} \int_{\mathbb{R}} \int_{\mathbb{R}} \int_{\mathbb{R}} \int_{\mathbb{R}} \overline{\psi_{a,b}^\alpha(w)} \overline{\chi_{a,b}^\alpha(v)} \phi_{a,b}^\alpha(u) h(w) g(v) |a|^{2\rho-3} da db dw dv. \end{aligned} \quad (2.3)$$

**Example 2.1. Basic function  $D^\alpha(u, v, w)$  for general novel fractional morlet wavelet transform**

Let  $\psi(t) = \chi(t) = \phi(t) = e^{i\omega_0 t - \frac{1}{2}t^2}$  be a morlet wavelet [2]. Then the general novel fractional morlet wavelet transform is given by  $\phi_{a,b}^\alpha(t) = |a|^{-\rho} e^{i\omega_0 \frac{(t-b)}{a} - \frac{1}{2}[\frac{(t-b)}{a}]^2}$ .

Now, from (2.2)

$$\begin{aligned}
D^\alpha(u, v, w) &= C_\phi^{-1} \int_{\mathbb{R}} \int_{\mathbb{R}} e^{(i/2)(w^2+v^2-u^2-b^2)\cot\theta} \overline{\psi_{a,b}(w)} \overline{\chi_{a,b}(v)} \phi_{a,b}(u) |a|^{2\rho-3} da db. \\
&= C_\phi^{-1} e^{(i/2)(w^2+v^2-u^2)\cot\theta} \int_{\mathbb{R}} \int_{\mathbb{R}} e^{(-i/2)b^2\cot\theta} e^{-iw_o \frac{(w-b)}{a} - \frac{1}{2} \frac{(w-b)^2}{a^2}} \\
&\quad \times e^{-iw_o \frac{(v-b)}{a} - \frac{1}{2} \frac{(v-b)^2}{a^2}} e^{iw_o \frac{(u-b)}{a} - \frac{1}{2} \frac{(u-b)^2}{a^2}} |a|^{\rho-3} db da \\
&= C_\phi^{-1} e^{(i/2)(w^2+v^2-u^2)\cot\theta} \int_{\mathbb{R}} \int_{\mathbb{R}} e^{(-i/2)b^2\cot\theta} \\
&\quad \times e^{(itw_o)(b+u-w-v)} e^{-t^2 \frac{(w-b)^2+(v-b)^2+(u-b)^2}{2}} |t|^{\rho+1} dt db \\
&= 2C_\phi^{-1} e^{(i/2)(w^2+v^2-u^2)\cot\theta} \int_{\mathbb{R}} \int_{\mathbb{R}} e^{(-i/2)b^2\cot\theta} \cos[w_o t(b+u-w-v)] \\
&\quad \times e^{-t^2 \frac{(w-b)^2+(v-b)^2+(u-b)^2}{2}} |t|^{\rho+1} dt db \\
&= C_\phi^{-1} e^{(i/2)(w^2+v^2-u^2)\cot\theta} \Gamma(1 + \rho/2) 2^{(1+\rho/2)} \\
&\quad \times \int_{\mathbb{R}} [(w-b)^2 + (v-b)^2 + (u-b)^2]^{-1-\rho/2} \\
&\quad \times e^{(-i/2)b^2\cot\theta} {}_1F_1(1 + \rho/2; 1/2; \frac{-w_o^2(b+u-w-v)^2}{2[(w-b)^2 + (v-b)^2 + (u-b)^2]}) db, \rho > 0,
\end{aligned}$$

by [4], p.15(14)] where  ${}_1F_1(a; b; u)$  is confluent hypergeometric function.

**Example 2.2. Basic function  $D^\alpha(u, v, w)$  for general novel fractional mexican hat wavelet transform**

The corresponding Mexican-Hat wavelet [2] is  $\psi(t) = \chi(t) = \phi(t) = (1 - t^2) e^{-\frac{t^2}{2}}$ . Then the general novel fractional mexican hat wavelet transform is given by  $\phi_{b,a}^\alpha(t) = |a|^{-\rho} \left(1 - \frac{(t-b)^2}{a^2}\right) e^{-\frac{(t-b)^2}{2a^2}}$ .

Now by using (2.2), we have

$$\begin{aligned}
D^\alpha(u, v, w) &= C_\phi^{-1} e^{(i/2)(w^2+v^2-u^2)\cot\theta} \int_{\mathbb{R}} \int_{\mathbb{R}} e^{(-i/2)b^2\cot\theta} \left(1 - \frac{(w-b)^2}{a^2}\right) \\
&\quad \times \left(1 - \frac{(v-b)^2}{a^2}\right) \left(1 - \frac{(u-b)^2}{a^2}\right) \\
&\quad \times e^{(-1/2a^2)((w-b)^2+(v-b)^2+(u-b)^2)} db da. \\
&= C_\phi^{-1} e^{(i/2)(w^2+v^2-u^2)\cot\theta} \int_{\mathbb{R}} e^{(-i/2)b^2\cot\theta} e^{(-t^2L/2)} t^{\rho+1} \\
&\quad \times (t^4 - Nt^2 + M) dt db. \\
&= C_\phi^{-1} e^{(i/2)(w^2+v^2-u^2)\cot\theta} \int_{\mathbb{R}} e^{(-i/2)b^2\cot\theta} db (-1/2) (-1)^{\rho+1} \\
&\quad \times \left( \Gamma((\rho+6)/2) (L/2)^{-(\rho+6)/2} - \Gamma((\rho+4)/2) N (L/2)^{-(\rho+4)/2} \right. \\
&\quad \left. + \Gamma((\rho+2)/2) M (L/2)^{\frac{-(\rho+2)}{2}} \right)
\end{aligned}$$

by [4], p.313(13) , where  $\rho > 0$ ,  $L = [(w-b)^2 + (v-b)^2 + (u-b)^2]$ ,

$M = [(u-b)^2(v-b)^2 + (u-b)^2(w-b)^2 + (v-b)^2(w-b)^2]$  and

$N = [L + (u-b)^2(v-b)^2(w-b)^2]$ .

In the following section we have obtained boundedness result for the basic function  $D^\alpha(u, v, w)$  and then establish existence theorem for the general novel fractional wavelet convolution and prove  $W_\phi^\alpha(h \#^\alpha g) = (W_\psi^\alpha h)(W_\chi^\alpha g)$ .

### 3. EXISTENCE THEOREMS

First we obtain boundedness results for the basic function  $D^\alpha(u, v, w)$ .

**Theorem 3.1.** *Let  $(1 + |z|^\rho)\phi(z) \in L^p(\mathbb{R})$ ,  $\chi \in L^q(\mathbb{R})$ ,  $\frac{1}{p} + \frac{1}{q} = 1$  and  $(1 + |z|^\rho)\psi(z) \in L^1(\mathbb{R})$ ,  $\rho \geq 0$ . Then*

$$\begin{aligned} |D^\alpha(u, v, w)| &\leq 2^{\rho+\frac{1}{p}} C_\phi^{-1} |v - w|^{-\frac{1}{q}} |u - w|^{-\frac{1}{p}-\rho} \|\chi\|_q \|(1 + |z|^\rho)\phi(z)\|_p \\ &\quad \times \|(1 + |z|^\rho)\psi(z)\|_1, \end{aligned} \quad (3.1)$$

where  $O < C_\phi = \int_{\mathbb{R}} \frac{|\hat{\phi}(\omega)|^2}{|\omega|} d\omega < \infty$ .

**Proof 3.** *From (2.2), we have*

$$\begin{aligned} |D^\alpha(u, v, w)| &= |C_\phi^{-1} \int_{\mathbb{R}} \int_{\mathbb{R}} |a|^{-\rho} \overline{e^{(-i/2)(w^2-b^2)\cot\theta}} \psi\left(\frac{(w-b)}{a}\right) |a|^{-\rho} \overline{e^{(-i/2)(v^2-b^2)\cot\theta}} \\ &\quad \times \chi\left(\frac{(v-b)}{a}\right) |a|^{-\rho} e^{(-i/2)(u^2-b^2)\cot\theta} \phi\left(\frac{(u-b)}{a}\right) |a|^{2\rho-3} db da| \\ &= C_\phi^{-1} \int_{\mathbb{R}} \int_{\mathbb{R}} \overline{\psi\left(\frac{(w-b)}{a}\right)} \overline{\chi\left(\frac{(v-b)}{a}\right)} \left| \phi\left(\frac{(u-b)}{a}\right) \right| |a|^{-\rho-3} db da \end{aligned} \quad (3.2)$$

By using Theorem 2.1 [6] we get the required result.

**Theorem 3.2.** (i) *Let  $\psi \in L^1(\mathbb{R})$ ,  $\phi \in L^p(\mathbb{R})$ ,  $\chi \in L^q(\mathbb{R})$ ,  $p, q, > 1$ ,  $0 < \rho < 1$  and  $\frac{1}{p} + \frac{1}{q} = 1 + \rho$ . Then*

$$\int_{\mathbb{R}} |D^\alpha(u, v, w)| dw \leq C_\phi^{-1} C(p, \rho) |u - v|^{-\rho} \|\psi\|_1 \|\phi\|_p \|\chi\|_q, \quad (3.3)$$

where  $C(p, \rho)$  is a constant.

(ii) *Let  $\psi \in L^1(\mathbb{R})$ ,  $(1 + |y|^{\rho-1})\phi(y) \in L^1(\mathbb{R})$ ,  $(1 + |y|^{\rho-1})\chi(y) \in L^1(\mathbb{R})$ , and  $\rho \geq 1$ . Then*

$$\begin{aligned} \int_{\mathbb{R}} |D^\alpha(u, v, w)| dw &\leq C_\phi^{-1} 2^{\rho-1} |u - v|^{-\rho} [\|\phi(x)x^{\rho-1}\|_1 \|\chi\|_1 + \|\chi(y)y^{\rho-1}\|_1 \|\phi\|_1] \\ &\quad \times \|\psi\|_1. \end{aligned} \quad (3.4)$$

**Proof 4.** From (3.2) and using Theorem 2.2 [6] the required results follows.

**Theorem 3.3.** (i). Let  $\psi \in L^p(\mathbb{R}), \phi \in L^1(\mathbb{R}), \chi \in L^q(\mathbb{R}), p, q, > 1, 0 < \rho < 1$  and  $\frac{1}{p} + \frac{1}{q} = 1 + \rho$ . Then

$$\int_{\mathbb{R}} |D^\alpha(u, v, w)| du \leq C_\phi^{-1} C(p, \rho) |v - w|^{-\rho} \|\psi\|_p \|\phi\|_1 \|\chi\|_q. \quad (3.5)$$

(ii). Let  $\phi \in L^1(\mathbb{R}), (1 + |x|^{\rho-1})\chi(x) \in L^1(\mathbb{R}), (1 + |x|^{\rho-1})\psi(x) \in L^1(\mathbb{R})$ , and  $\rho \geq 1$ . Then

$$\begin{aligned} \int_{\mathbb{R}} |D^\alpha(u, v, w)| du &\leq C_\phi^{-1} 2^{\rho-1} |v - w|^{-\rho} [\|\psi(x)x^{\rho-1}\|_1 \|\chi\|_1 + \|\chi(x)x^{\rho-1}\|_1 \|\psi\|_1] \\ &\quad \times \|\phi\|_1. \end{aligned} \quad (3.6)$$

The proof is similar to that Theorem 3.2 .

**Theorem 3.4.** (i). Let  $\psi \in L^q(\mathbb{R}), \phi \in L^p(\mathbb{R}), \chi \in L^1(\mathbb{R}), p, q, > 1, 0 < \rho < 1$  and  $\frac{1}{p} + \frac{1}{q} = 1 + \rho$ . Then

$$\int_{\mathbb{R}} |D^\alpha(u, v, w)| dv \leq C_\phi^{-1} C(p, \rho) |u - w|^{-\rho} \|\psi\|_q \|\phi\|_p \|\chi\|_1. \quad (3.7)$$

(ii). Let  $\chi \in L^1(\mathbb{R}), (1 + |x|^{\rho-1})\phi(x) \in L^1(\mathbb{R}), (1 + |x|^{\rho-1})\psi(x) \in L^1(\mathbb{R})$ , and  $\rho \geq 1$ . Then

$$\begin{aligned} \int_{\mathbb{R}} |D^\alpha(u, v, w)| dv &\leq C_\phi^{-1} 2^{\rho-1} |u - w|^{-\rho} [\|\psi(x)x^{\rho-1}\|_1 \|\phi\|_1 + \|\phi(x)x^{\rho-1}\|_1 \|\psi\|_1] \\ &\quad \times \|\chi\|_1. \end{aligned} \quad (3.8)$$

The proof is similar to that Theorem 3.2

**Theorem 3.5.** Let  $\phi \in L^1(\mathbb{R}), \psi \in L^p(\mathbb{R}), \chi \in L^q(\mathbb{R}), p, q > 1, 0 < \rho < 1$ ,

$\frac{1}{p} + \frac{1}{q} = \rho + 1, h \in L^r(\mathbb{R})$  and  $g \in L^{r'}(\mathbb{R}), r, r' > 1, \frac{1}{r} + \frac{1}{r'} + \rho = 2$ . Then



$$\|(h \#^\alpha g)\|_1 \leq C_\phi^{-1} C(\rho, p, r) \|\phi\|_1 \|\psi\|_p \|\chi\|_q \|g\|_{r'} \|h\|_r$$

where  $C_\phi$  is given by (1.4) and  $C(\rho, p, r)$  is a constant.

**Proof 5.** we have

$$\begin{aligned} \int_{\mathbb{R}} |(h \#^\alpha g)(u)| du &\leq \int_{\mathbb{R}} \left( \int_{\mathbb{R}} |h^*(u, v)| |g(v)| dv \right) du \\ &= \int_{\mathbb{R}} |g(v)| dv \int_{\mathbb{R}} |h^*(u, v)| du \\ &\leq \int_{\mathbb{R}} |g(v)| dv \int_{\mathbb{R}} \left( \int_{\mathbb{R}} |D^\alpha(u, v, w)| |h(w)| dw \right) du \end{aligned}$$

by using above Theorem 3.3, we get

$$\int_{\mathbb{R}} |(h \#^\alpha g)(u)| du \leq C_\phi^{-1} C(p, \rho) \|\phi\|_1 \|\chi\|_q \|\psi\|_p \int_{\mathbb{R}} |g(v)| dv \int_{\mathbb{R}} |h(w)| |v - w|^{-\rho} dw$$

Therefore, by using Theorem 2.5 [6], we get the required result.

**Theorem 3.6.** Let  $\phi \in L^1(\mathbb{R}) \cap L^\infty(\mathbb{R})$ ,  $\chi \in L^q(\mathbb{R})$ ,  $\psi \in L^p(\mathbb{R})$ ,  $p, q > 1$ ,  $0 < \rho < 1$ ,  $\frac{1}{p} + \frac{1}{q} = \rho + 1$ . Assume further that  $h \in L^r(\mathbb{R})$ ,  $g \in L^{r'}(\mathbb{R})$ ,  $r, r' > 1$  and  $\frac{1}{r} + \frac{1}{r'} + \rho = 2$ . Then

$$W_\phi^\alpha(h \#^\alpha g)(a, b) = (W_\psi^\alpha h)(a, b)(W_\chi^\alpha g)(a, b)$$

**Proof 6.** By using above Theorem 3.5,  $(h \#^\alpha g) \in L^1(R)$ . As given basic wavelet  $\phi \in L^1(R) \cap L^\infty(R)$ ,  $W_\phi^\alpha(h \#^\alpha g)(a, b)$  exist.

From (2.1), we have

$$\begin{aligned}
W_\phi^\alpha(h\#^\alpha g)(a, b) &= \int_{\mathbb{R}} (h\#^\alpha g)(u) \overline{\phi^\alpha\left(\frac{u-b}{a}\right)} |a|^{-\rho} du \\
&= \int_{\mathbb{R}} \overline{\phi^\alpha\left(\frac{u-b}{a}\right)} du \int_{\mathbb{R}} \int_{\mathbb{R}} D^\alpha(u, v, w) h(w) g(v) |a|^{-\rho} dw dv \\
&= \int_{\mathbb{R}} \int_{\mathbb{R}} h(w) g(v) dw dv \int_{\mathbb{R}} D^\alpha(u, v, w) \overline{\phi^\alpha\left(\frac{u-b}{a}\right)} |a|^{-\rho} du \\
&= \int_{\mathbb{R}} \int_{\mathbb{R}} h(w) g(v) dw dv \overline{\psi_{a,b}^\alpha(w)} \overline{\chi_{a,b}^\alpha(v)} \\
&= \int_{\mathbb{R}} h(w) \overline{\psi_{a,b}^\alpha(w)} dw \int_{\mathbb{R}} g(v) \overline{\chi_{a,b}^\alpha(v)} dv \\
&= (W_\psi^\alpha h)(a, b) (W_\chi^\alpha g)(a, b)
\end{aligned}$$

#### ACKNOWLEDGMENT

The work of the first author was supported by U.G.C start-up grant for newly recruited faculty.

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